Lenz's Law and Motional EMF

Objectives

After going through this lesson, the learners will be able to:

- Understand Lenz's law
- Relate Lenz's law and the law of conservation of energy
- Use Fleming's right hand rule to predict the direction of induced emf /current
- Know different ways of producing an induced emf in a conductor
- Recognise and derive an expression for Motional emf

Content Outline

- Unit Syllabus
- Module Wise Distribution of Unit Syllabus
- Words You Must Know
- Introduction
- Lenz's Law and Energy Conservation
- Fleming's Right Hand Rule
- Various Methods of Producing Induced EMF
- Motional EMF
- Energy Consideration in Motional EMF
- Summary

Unit Syllabus

Unit IV: Electromagnetic Induction and Alternating Currents:

Chapter-6: Electromagnetic Induction

Electromagnetic induction; Faraday's laws, induced emf and current; Lenz's Law, Eddy currents; Self induction and mutual induction.

Chapter-7: Alternating Current

Alternating currents, peak and rms value of alternating current/voltage; reactance and impedance; LC oscillations (qualitative treatment only), LCR series circuit, resonance; power in AC circuits, wattless current. AC generator and transformer.

Module Wise Distribution of Unit Syllabus - 9 Modules

The above unit is divided into 9 modules for better understanding.

Module 1	• Electromagnetic induction • Faraday's laws, induced emf and current; • Change of flux • Rate of change of flux
Module 2	• Lenz's Law, • Conservation of energy • Motional EMF
Module 3	• Eddy currents. • Self induction • Mutual induction. • Unit • Numerical
Module 4	• AC generator • Alternating currents, • Representing ac • Formula • Graph • Phasor • Frequency of ac and what does it depend upon • peak and rms value of alternating current/voltage;
Module 5	• AC circuits • Components in ac circuits • Comparison of circuit component in ac circuit with that if used in dc circuit • Reactance mathematically • Pure R • Pure L

	• Pure C • Phasor, graphs for each
Module 6	• AC circuits with RL, RC and LC components • Impedance; LC oscillations (qualitative treatment only), • Resonance • Quality factor
Module 7	• Alternating voltage applied to series LCR circuit • Impedance in LCR circuit • Phasor diagram • Resonance • Power in ac circuit • Power factor • Wattles current
Module 8	• Transformer
Module 9	• Advantages of ac over dc • Distribution of electricity to your home

Module 2

Words You Must Know

Let us remember the words we have been using in our study of this physics course:

- **Magnetic Field:** The region around a magnet, within which its influence can be felt.
- **Electromotive Force:** The amount of work done by an external source, to take a unit positive charge once round the circuit.
- **Area Vector:** A vector perpendicular to a given area whose magnitude is equal to the given area.
- **Electromagnetic Induction:** An EMF is induced in a conductor whenever a magnetic flux changes around it.
- **Magnetic Flux:** Just like electric flux, magnetic flux Φ_B through any surface of area **A** held perpendicularly in magnetic field **B** is given by the total number of magnetic lines of force crossing the area. Mathematically, it is equal to the dot product of **B** and **A**.

  $\Phi_B = \mathbf{B}.\mathbf{A} = BA\cos\theta$, where θ is the angle between **B** and **A**

- **Induced EMF and Induced Current:** The emf developed in a loop when the magnetic flux linked with it changes with time is called induced emf when the conductor is in the form of a closed loop, the current induced in the loop is called an induced current.
- **Weber:** One weber is defined as the amount of magnetic flux, through an area of $1m^2$ held normal to a uniform magnetic field of one Tesla. The SI unit of magnetic flux is weber (Wb) or tesla metre squared (Tm^2).
- **Faraday's Laws of Electromagnetic Induction:**
    - **First Law:** It states that whenever the amount of magnetic flux linked with the coil changes with time, an emf is induced in the coil. The induced emf lasts in the coil only as long as the change in the magnetic flux continues.
    - **Second Law:** It states that the magnitude of the emf induced in the coil is directly proportional to the time rate of change of the magnetic flux linked with the coil.

Introduction

We have studied in module 1 of this unit that emf can be induced in a conductor if the magnetic flux linked with the conductor changes with time. This establishes a link between electricity and magnetism.

To a lay man, it may appear that we are generating electrical energy out of nowhere !! This definitely cannot be true, for it would violate the law of conservation of energy. The questions that arise are:

- **Where does the electrical energy associated with the induced emf or current come from?**
- **Which energy is getting converted into electrical energy?**
- **Is there any specific direction in which current will be induced?**
- **How can we determine that direction?**

In this module, we will try to answer the above questions. Along with that we will also study various methods of inducing emf.

Let us begin with **Lenz's law; it is this law that helps us to understand the source; or cause of generation of electrical energy during the phenomenon of electromagnetic induction.**

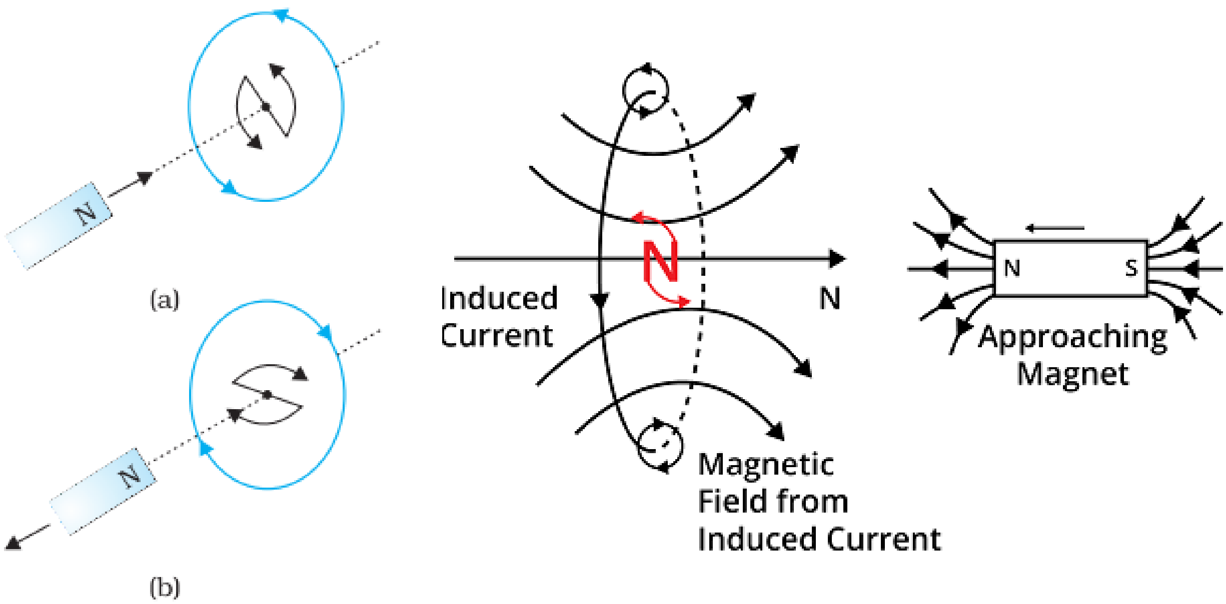
Lenz's Law and Energy Conservation

In **1834,** German physicist Heinrich Friedrich Lenz (1804-1865), gave a rule, known as Lenz's law which gives the 'polarity of the induced emf, in a clear and concise way'.

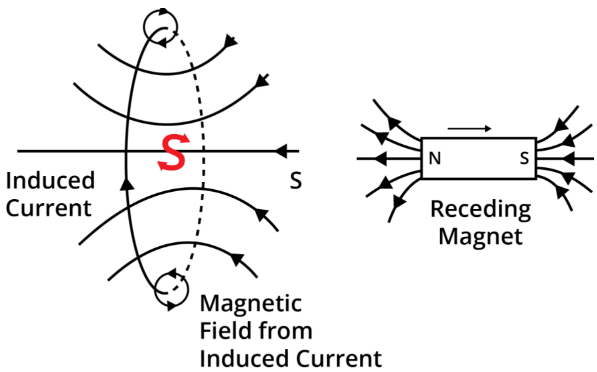
The law states: "**The direction of induced emf is always such that it opposes the change in magnetic flux responsible for its production**".

As the North-pole of a bar magnet moves towards the coil, the magnetic flux through the coil increases**. Hence current would be induced in the coil, in such a direction, that it opposes this increase in flux**. **This is possible only if the current in the coil is in a counter-clockwise direction with respect to an observer situated on the side of the magnet, this means the coil acquires a north polarity on its face towards which the North pole of the magnet is approaching.**

Illustration of Lenz's law.



Or



The Question is Why?

Suppose that the induced current was in the direction opposite to the one suggested, that is it acquires south polarity. In that case, the South-pole due to the induced current will face the approaching North-pole of the magnet. The bar magnet will then be attracted towards the coil at an ever increasing acceleration. A gentle push on the magnet will initiate the process and its velocity and kinetic energy will continuously increase without expending any energy. If this could happen, one could construct a perpetual-motion machine by a suitable arrangement.

This violates the law of conservation of energy and hence cannot happen.

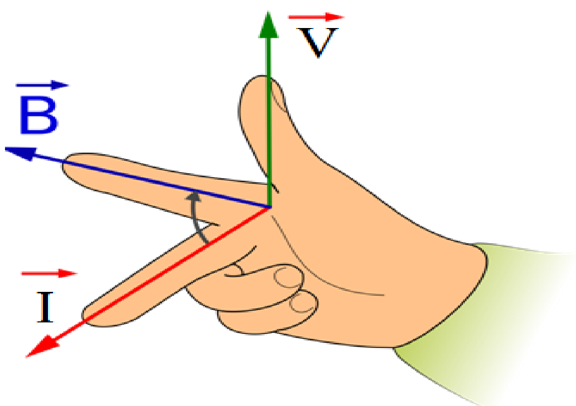
In the correct situation as suggested by Lenz's law, the approaching bar magnet experiences a repulsive force due to the induced current. Therefore, some external agent has to do work for moving the magnet towards the coil. It is this mechanical work done in moving the magnet with respect to the coil that changes into electrical energy, through the induced emf/current.

When there is no relative motion between the magnet and the coil there is no work done and hence no induced emf/current.

Fleming's Right Hand Rule

Fleming's right hand rule gives us **the direction of induced emf / current in a conductor moving in a magnetic field.**

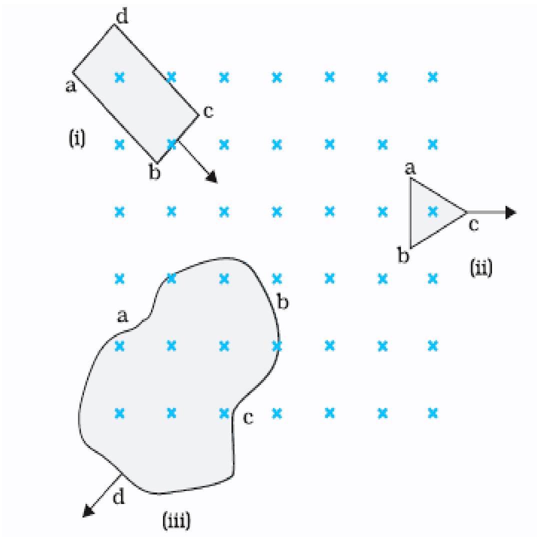
If we stretch the fore-finger, central finger and thumb of our right hand, mutually perpendicular to each other such that the **fore-finger** is in the **direction of the magnetic field** , the **thumb** is in the **direction of motion of the conductor** , then the **central finger** would give the **direction of the induced current.**



Example

Figure shows planar loops of different shapes moving out of or into a region of a magnetic field which is directed normal to the plane of the loop away from the reader. Determine the direction of induced current in each loop using Lenz's law.

Solution



(i) The magnetic flux through the rectangular loop abcd increases, due to the motion of the loop into the region of magnetic field, the induced current must flow along the path **b c d a b** so that it opposes the increasing flux.

(ii) Due to the outward motion, magnetic flux through the triangular loop abc decreases due to which the induced current flows along **b a c b,** so as to oppose the change in flux.

(iii)As the magnetic flux decreases due to motion of the irregular shaped loop abcd out of the region of magnetic field, the induced current flows along **c d a b c,** so as to oppose change in flux.

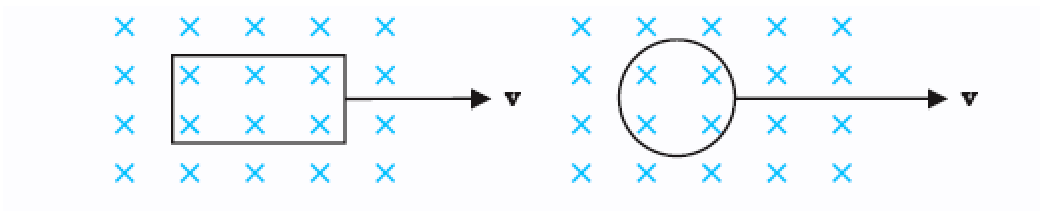
Note that there are no induced current as long as the loops are completely inside or outside the region of the magnetic field

Example

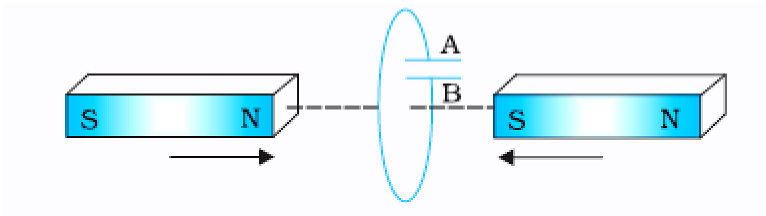
1. A closed loop is held stationary in the magnetic field between the north and south poles of two permanent magnets held fixed. Can we hope to generate current in the loop by using very strong magnets?

2. A closed loop moves normally to the constant electric field between the plates of a large capacitor. Is a current induced in the loop?
    i. When it is wholly inside the region between the capacitor plates
    ii. When it is partially outside the plates of the capacitor? The electric field is normal to the plane of the loop.

3. A rectangular loop and a circular loop are moving out of a uniform magnetic field region to a field-free region with a *constant velocity* v. In which loop do you expect the induced emf to be constant *during* the passage out of the field region?
   The field is normal to the loop



Predict the polarity of the capacitor in the situation described by the fig



Solution

(a) **No**. However strong the magnet may be, current can be induced only by changing the magnetic flux through the loop.

(b) **No** current is induced in either case. Current cannot be induced by changing the electric flux.

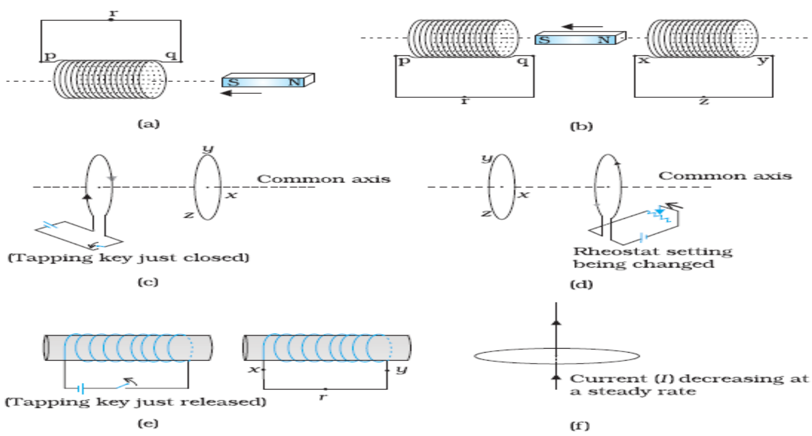
(c) The induced EMF is expected to be constant only in the case of the rectangular loop. In the case of a circular loop, the rate of change of area of the loop during its passage out of the field region is not constant; hence induced *emf* will vary accordingly.

(d) The polarity of plate 'A' will be positive with respect to plate 'B' in the capacitor.

Problems for Practice

Example

Predict the direction of induced current in the situations described by the following Figs. (a) to (f).



Answer

a) Along qrpq
b) Along prq, along yzx
c) Along yzx

d) Along zyx

e) Along xry

f) No induced current since field lines lie in the plane of the loop

Example

A horizontal straight wire 10 m long extending from east to west is falling with a speed of 5.0 m s^{-1} at right angles to the horizontal component of the earth's magnetic field, 0.30×10^{-4} Wb m^{-2}.

i. What is the instantaneous value of the emf induced in the wire?

ii. What is the direction of the EMF?

iii. Which end of the wire is at the higher electrical potential?

Answer

i. 1.5×10^{-3} V

ii. West to east

iii. Eastern end

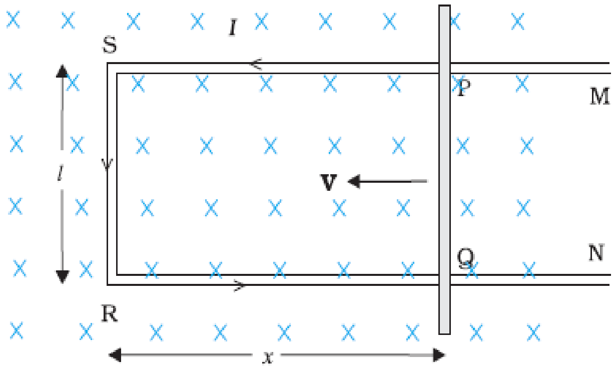
Various Methods of Producing Induced EMF

An emf will be induced in a circuit whenever the magnetic flux linked with the circuit changes. We know that the magnetic flux is given by Ø = B.A = BAcosθ. Hence, Ø can be changed by changing B, A or θ individually or by changing any two or all three of them, simultaneously. Therefore, there are three methods of generating an inducing emf:

- **Induced EMF by changing the magnetic field:** In the first and third experiment, conducted by Faraday, the movement of the magnet or the pressing of the key of coil results in a change in the magnetic field linked with the coil, this induces an emf.
- **Induced EMF by changing the orientation of coil with respect to the magnetic field:** When the coil rotates in a magnetic field the angle θ changes hence the magnetic flux linked with the coil changes and this induces an emf. This is the basis of the ac generator.
- **Induced EMF by changing the area A (MOTIONAL EMF):** Motional EMF is a type of induced emf which occurs when a wire is pulled through the magnetic field.

The magnitude of motional emf depends upon the velocity of the wire, strength of the magnetic field and the length of the wire. Here, in effect, the area A, in the expression for magnetic flux, is being made to change with time.

Motional EMF



Let us consider a straight conductor moving in a uniform and time independent magnetic field. The figure shows a conductor NRSM and a conducting rod PQ moving on the conductor. Let the rod PQ move towards the left with a constant velocity v as shown in the figure.

Assume that there is no loss of energy due to friction. PQRS then forms a closed circuit enclosing an area that changes as PQ moves. It is placed in a uniform magnetic field B which is perpendicular to the plane of this system and is directed into the screen If the length RQ = *x* and SR = *l*, the magnetic flux $\emptyset_B$ enclosed by the loop PQRS will be:

$$\emptyset B = B.A = B\,l\,x$$

Since, x is changing with time, the rate of change of this flux is given by:

$$\frac{d\emptyset_B}{dt} = \frac{d}{dt}(Blx) = B\,l\,\frac{dx}{dt}$$

Hence, the induced emf is given by:

$$\epsilon = -\frac{d\emptyset_B}{dt} = Bl\left(-\frac{dx}{dt}\right) = B\,l\,v \text{........} (1) \left\{as,\ v = -\frac{dx}{dt}\right\}$$

The induced emf B*l*v is called motional emf.

Note

We are able to produce an induced emf by moving a conductor instead of varying the magnetic field. Here, we are changing the magnetic flux, linked with the circuit, by changing its (effective) area enclosing the magnetic flux.

Example

A jet plane is travelling towards west at a speed of $500 ms^{-1}$.What is the voltage difference developed between the ends of the wing having a span of 25 m, if the Earth's magnetic field at the location has a magnitude of 5×10^{-4} T and the dip angle is 30°.

Solution

Vertical component of B = $5 \times 10^{-4} \sin 30^{\circ} = 2.5 \times 10^{-4}$ T

e =B l v

e =$2.5 \times 10^{-4} \times 25 \times 500 = 3.125$ V

Motional EMF arises due to the motion of charges through a magnetic field.

As the rod of length l is moved with a velocity v, in a uniform magnetic field, each charge within the rod moves with this velocity v and experiences a force F = q v B.

The mobile free electrons in the rod are driven in the rod from P to Q according to Fleming's Left hand rule.

Here, the Lorentz force on this charge is q v B in magnitude, and its direction is towards Q.

All charges experience the same force, in magnitude and direction, irrespective of their position in the rod PQ. The work done in moving a charge q from P to Q is

$$W = F \times l$$

$$= q\,v\,B\,l$$

The induced EMF is equal to work done per unit charge. Hence,

$$\varepsilon = \frac{W}{q}$$

$$= \frac{qvBl}{q}$$

$$= B\,l\,v$$

This equation gives the emf induced across the rod PQ.

This helps us to understand the cause of production of the motional EMF.

It gives us a way of understanding Faraday’s law when a conductor is moving in a uniform and time-independent magnetic field.

What happens when the conductor is not in motion but is placed in a time dependent magnetic field?

Faraday showed experimentally that an emf is developed even in this case.

We will now discuss this case.

You will recall that the force on the charges, in a stationary conductor, is given by:

Lorentz force on a charge q is given by

$$F = q\,E + q\,(v \times B)$$

Since the conductor is stationary, any force on the charges must arise because of Electric field term E alone.

Hence, F = q E

Therefore, to explain the existence of induced emf and induced current, in this case we need to assume that a time-varying magnetic field generates an electric field.

Here, we also need to add that electric fields, produced by static electric charges can have properties different from those produced by time-varying magnetic fields.

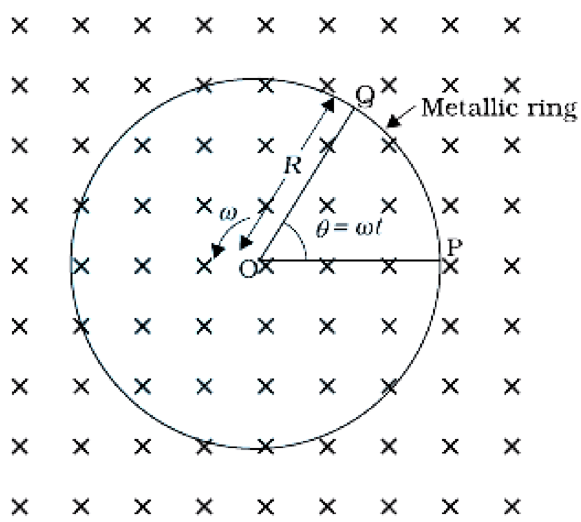
We Have Learnt That

- **Charges in motion (current) can exert force/torque on a stationary magnet. Conversely,**
- **A bar magnet in motion (or more generally, a changing magnetic field) can exert a force on the stationary charge.**

This is the fundamental significance of Faraday's discovery.Electricity and magnetism are related to each other. **Let us discuss with special examples**

Example

A metallic rod of 1 m length is rotated with a frequency of 50 rev/s, with its one end hinged at the centre and its other end at the circumference of a circular metallic ring of radius 1 m. The axis of rotation is an axis, passing through the centre and perpendicular to the plane of the ring. A constant and uniform magnetic field of 1T parallel to the axis is present everywhere. Find the induced emf between the centre and the metallic ring.



Solution

As the rod is rotated, free electrons in the rod move towards the outer end due to Lorentz force and get distributed over the ring. Thus, the resulting separation of charges produces an emf across the ends of the rod. At a certain value of emf, there is no more flow of electrons and a steady state is reached. The magnitude of the emf generated across a length d*r* of the rod, as it moves at right angles to the magnetic field is given by:

$d\varepsilon = B\, v\, dr$.

Hence,

$$\epsilon = \int d\epsilon = \int Bvdr = \int B\omega r dr \qquad [\because v = \omega r]$$

$$= \frac{B\omega r^2}{2}$$

$$\epsilon = \frac{1}{2} \times 1 \times 2\pi \times 50 \times 1^2 \quad (\because \omega = 2\pi v)$$

$$= 157 \text{ V}$$

Example

A wheel with 10 metallic spokes each 0.5 m long is rotated with a speed of 120 rev/min in a plane normal to the horizontal component of earth's magnetic field H_E at a place.

If H_E= 0.4 G at the place, what is the induced emf between the axle and the rim of the wheel? Note that 1 G = 10^{-4} T.

Solution

$$\text{Induced emf} = \frac{1}{2}\omega BR^2$$

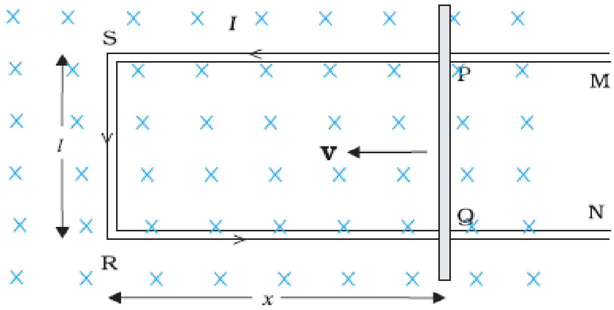
$$= \frac{1}{2} \times 4\pi \times 0.4 \times 10^{-4} \times (0.5)^2$$

$$= 6.28 \times 10^{-5} \text{ V}$$

The number of spokes is immaterial because the emf's across the spokes are in parallel.

Energy Consideration in Motional EMF

Let *r* be the resistance of the movable arm PQ of the rectangular conductor shown in the Figure.



We assume that the remaining arms QR, RS and SP have resistances that are negligible compared to *r*. Thus, the overall resistance of the rectangular loop is *R* and this does not change as PQ is moved. The current I in the loop:

$$I = \frac{e}{R} = \frac{B\,l\,v}{R}$$

The magnitude of force on the conductor PQ moving in the magnetic field is given by:

$$F = BIl = B\left(\frac{B\,l\,v}{R}\right)l = \frac{B^2l^2v}{R}$$

The direction of the force is opposite to the velocity of the conductor.

Hence, the power required to keep the conductor with velocity v is given by:

$$P = F \times v$$

$$= \frac{B^2l^2v}{R} \times v$$

$$= \frac{B^2l^2v^2}{R}$$

The external agent that does this work is doing mechanical work. Where does this mechanical energy go?

The answer is: It is dissipated as Joule heat, which is given by:

$$P = I^2R = \left(\frac{B\,l\,v}{R}\right)^2 R = \frac{B^2l^2v^2}{R}$$

Hence, the mechanical energy required to move the conductor PQ is converted first to electrical energy (induced emf) and then to thermal energy.

There is an interesting relation between the total charge that flows through the circuit and the change in the magnetic flux linked with it.

From Faraday's law, we have learnt that the magnitude of the induced emf is:

$$e = \frac{d\phi}{dt}$$

Comparing the two equations we get:

$d\emptyset = dQ\,R$

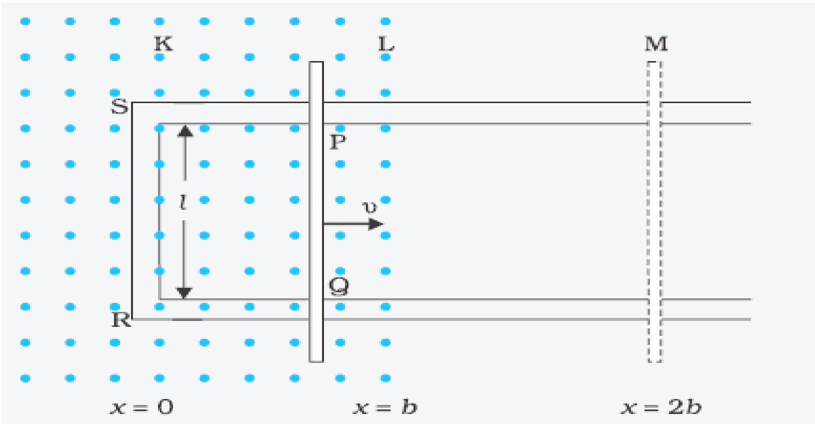
$dQ = \frac{d\emptyset}{R}$

This leads to Q = $\frac{change\ in\ magnetic\ flux}{resistance}$

Let us use this in the following example.

Example

The arm PQ of the rectangular conductor is moved from x = 0, outwards. The uniform magnetic field is perpendicular to the plane and extends from x = 0 to x = b and is zero for x > b. Only the arm PQ possesses substantial resistance 'r'. Consider the situation when the arm PQ is pulled outwards from x = 0 to x = 2b, and is then moved back to x = 0 with the same constant speed v.



Obtain expressions for the flux, the induced emf, the force necessary to pull the arm and the power dissipated as Joule heat.Sketch the variation of these quantities with distance.

Solution

Let us first consider the forward motion from x = 0 to x = 2b

The flux Φ_B linked with the circuit SPQR is:

$\Phi_B = B\,l\,x \qquad 0 \leq x < b \qquad [\emptyset = BA \text{ and } A = lx]$

$= B\,l\,b \qquad b \leq x < 2b \qquad$ [here only A=l b is effective as for x>b, B=0]

The induced EMF is,

$e = -\frac{d\emptyset}{dt}$

$= -B\,l\,v \qquad 0 \leq x < b \qquad \left[\frac{dx}{dt} = v\right]$

$= 0 \qquad b \leq x < 2b$ [B l b is a constant and derivative of a constant is zero]

When the induced emf is non-zero, the current I is (in magnitude):

$I = B\,l\,v \qquad 0 \le x < b$

Also,

$I = 0 \qquad b \le x < 2b \quad [\mathcal{E} = 0]$

The force required to keep the arm PQ in constant motion is I *l* B. Its direction is to the left. In magnitude:

$F = I\,l\,B$

$$F = \frac{B\,l\,v}{r} \times l \times B = \frac{B^2 l^2 v}{r} \qquad 0 \le x < b$$

$F = 0 \qquad b \le x < B \quad [I = 0]$

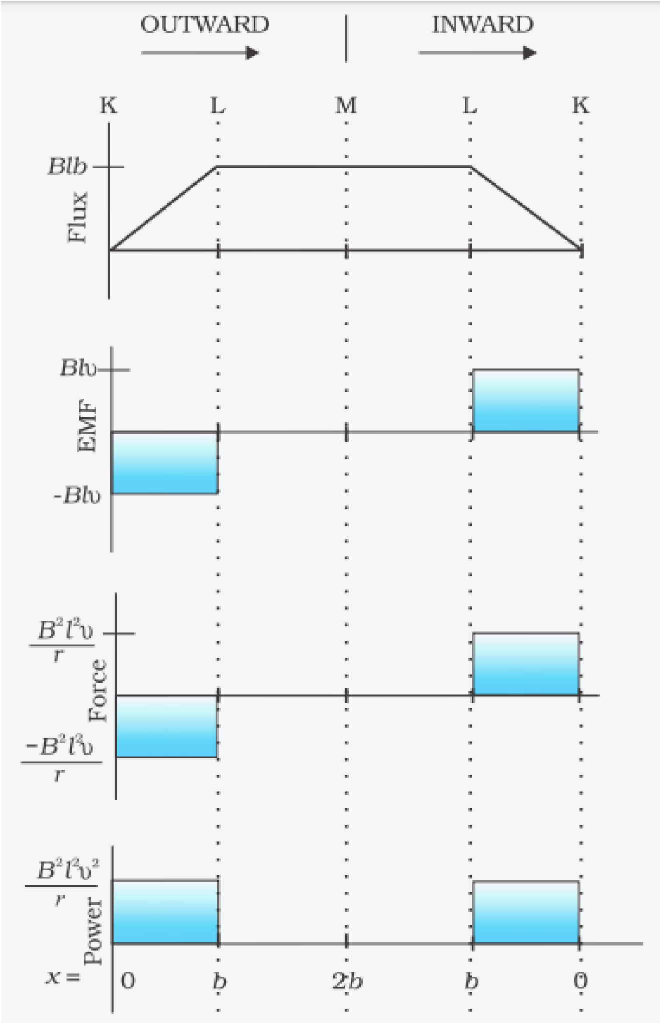
The Joule heating loss is:

$$P_j = F\,v = I^2 R$$

$$= \frac{B^2 l^2 v^2}{r} \qquad 0 \le x < b$$

$$= 0 \qquad b \le x < 2b \qquad [F = 0]$$

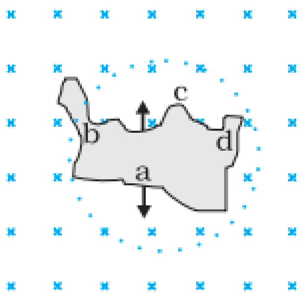
One obtains similar expressions for the inward motion from x = 2b to x = 0. One can appreciate the whole process by examining the sketch of various quantities displayed below:



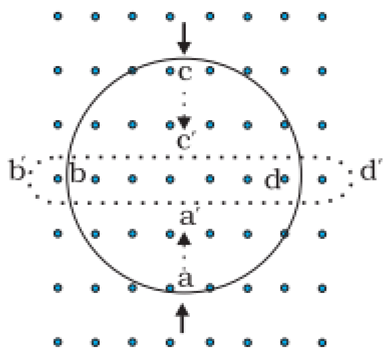
Shows the whole process, examine the sketch of various quantities

Try These

- **Use Lenz's law to determine the direction of induced current in the situations described by**
  - **a. A wire of irregular shape turning into a circular shape;**



  - **b. A circular loop being deformed into a narrow straight wire.**



- **A rectangular wire loop of sides 8 cm and 2 cm with a small cut is moving out of a region of uniform magnetic field of magnitude 0.3 T directed normal to the loop. What is the emf developed across the cut if the velocity of the loop is 1 cm s^{-1} in a direction normal to the (a) longer side, (b) shorter side of the loop?**
  For how long does the induced voltage last in each case?

- **A 1.0 m long metallic rod is rotated with an angular frequency of 400 rad s^{-1} about an axis normal to the rod passing through its one end. The other end of the rod is in contact with a circular metallic ring. A constant and uniform magnetic field of 0.5 T parallel to the axis exists everywhere.**
  Calculate the emf developed between the centre and the ring.

- **A circular coil of radius 8.0 cm and 20 turns is rotated about its vertical diameter with an angular speed of 50 rad s^{-1} in a uniform horizontal magnetic field of magnitude 3.0×10^{-2} T. Obtain the maximum and average emf induced in the coil. If the coil forms a closed loop of resistance 10 Ω, calculate the maximum value of current in the coil.**
  Calculate the average power loss due to Joule heating.
  Where does this power come from?

- **A horizontal straight wire 10 m long extending from east to west is falling with a speed of 5.0 ms^{-1}, at right angles to the horizontal component of the earth's magnetic field, 0.30×10^{-4} Wb m^{-2}.**
  a. **What is the instantaneous value of the emf induced in the wire?**
  b. **What is the direction of the emf?**
  c. **Which end of the wire is at the higher electrical potential?**

Summary

- **Lenz's Law:** The law states that the direction of induced emf is always such that it opposes the change in magnetic flux responsible for its production.
- Lenz's law states that the polarity of the induced emf is such that it tends to produce a current which opposes the change in magnetic flux that produces it. The negative sign in the expression for Faraday's law indicates this fact.
- **Fleming's Right Hand Rule:** Fleming's right hand rule gives us the direction of induced emf/current in a conductor moving in a magnetic field.

  If we stretch the fore-finger, central finger and thumb of our right hand mutually perpendicular to each other such that the fore-finger is in the direction of the field **,** the thumb is in the direction of motion of the conductor, then the central finger would give the direction of the induced current.
- **Induced EMF by Changing The Magnetic Field:** The movement of magnet or pressing the key of coil results in changing the magnetic field associated with the coil, this induces the emf.
- **Induced EMF by Changing The Orientation of Coil and Magnetic Field:** When the coil rotates in a magnetic field the angle Θ changes and magnetic flux linked with the coil changes and this induces the emf. This is the basis of ac generators.
- **Induced EMF by Changing The Area A: MOTIONAL EMF:** Motional emf is a type of induced emf which occurs when a wire is pulled through the magnetic field. The magnitude of motional emf depends upon the velocity of the wire, strength of magnetic field and the length of the wire. When a metal rod of length l is placed normal to a uniform magnetic field B and moved with a velocity v perpendicular to the field, the induced emf (called motional emf) across its ends is $\varepsilon = Blv$